Midterm Test in MAT 410: Introduction to Topology

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Date and time: Friday, December 18, 2009, 18:00 - 19:00.

NAME:

You have **60 minutes** of time to answer the 12 questions below. You are not allowed to use anything else than a pen and blank sheets of paper. The rules announced by e-mail apply.

Question 1: Give the definition of a *topological space*. (3 credits)

Question 2: Give the definition of a *metric space*. (3 credits)

Question 3: Let $X := \{1, 2, 3, 4\}$ be a set of cardinality 4, and define a topology on X by letting the open sets be $\{4\}$, $\{1, 2\}$, $\{1, 2, 3\}$ and $\{2, 3, 4\}$. Either prove that X with this collection of open sets is a topological space, or list at least 4 violations of the axioms of a topological space. (4 credits)

Question 4: Let $X := \{1, 2, 3, 4\}$ be a set of cardinality 4, and let $d : X \times X \to \mathbb{R}_0^+$ be the mapping defined by $d(1,1) = 0$, $d(1,2) = 1$, $d(1,3) = 6$, $d(1,4) = 2$, $d(2,1) = 2$, $d(2,2) = 0$, $d(2,3) = 4$, $d(2,4) = 1$, $d(3,1) = 6$, $d(3,2) = 4$, $d(3,3) = 2$, $d(3,4) = 2$, $d(4,1) = 2$, $d(4,2) = 1$, $d(4,3) = 2$ and $d(4,4) = 0$. Either prove that (X, d) is a metric space, or list at least 4 violations of the axioms of a metric space. (4 credits)

Question 5: When is a topological space said to be a *Hausdorff space*? (2 credits)

Question 6: Let $X := \{1, 2, 3, 4\}$. Give an example of a topology with which X becomes a Hausdorff space. (2 credits)

Question 7: Let $X_1 := \{1, 2, 3\}$ and $X_2 := \{4, 5, 6\}$ be topological spaces, where the open sets in X_1 are $\emptyset$, $\{1, 2\}$, $\{3\}$ and X_1, and the open sets in X_2 are $\emptyset$, $\{4, 6\}$, $\{5\}$ and X_2. Give an example of a homeomorphism from X_1 to X_2. (2 credits)

Question 8: When is a metric space X said to be *complete*? (2 credits)

Question 9: Is a topological space always also a metric space? (2 credits)

Question 10: Give an example of a topological space in which the union of infinitely many closed sets is not always closed. (2 credits)

Question 11: The *Cantor set* is the subset of the unit interval $[0, 1]$ containing precisely those numbers which have a ternary numeral involving only digits 0 and 2. (Ternary digits are digits with respect to base 3, just like decimal digits are digits with respect to base 10). Determine the closure and the interior of the Cantor set in the usual topology on $\mathbb{R}$. (2 credits)

Question 12: Which problem occurs when trying to embed the Klein bottle into $\mathbb{R}^3$? (2 credits)

– Good luck!

Maximum possible number of credits: 30.
Grade = (number of credits)/3, rounded to the nearest integer.