# Introduction to Topology, Exercise Sheet 1

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Due: October 23, 2009

Exercise 1: Determine the supremum of the set $\{1 - x^2 - 2x^4 - x^6 \mid x \in \mathbb{R}\} \subset \mathbb{R}$. (1 credit)

Exercise 2: Compute

1.
$$\lim_{n \to \infty} \sum_{k=1}^{n} \frac{1}{2^k}.$$

2.
$$\lim_{n \to \infty} \frac{n^3 + 3n - 7}{n^3 - 10n + 11}.$$

(2 credits)

Exercise 3: Give an example of a bounded sequence $(a_n) \subset [0, 1]$ which for every $x \in [0, 1]$ has a subsequence converging to $x$. (2 credits)

Exercise 4: Determine the set of limit points of

1. $\mathbb{Q}$.

2. $\{a/2^b \mid a, b \in \mathbb{N}_0\}$.

3. $\{x \in \mathbb{R} \setminus \{0\} \mid \sin(1/x) = 0\}$.

(3 credits)

Exercise 5: Give an example of a function $f : \mathbb{R} \to \mathbb{R}$ which is nowhere continuous. (1 credit)

Exercise 6: Give an example of a continuous function $[0, 1[ \to \mathbb{R}$ which is not bounded. (1 credit)