Introduction to Topology, Exercise Sheet 2

Stefan Kohl

October 26, 2009

Due: November 4, 2009

Exercise 7: Derive formulas for computing the area of the union and the intersection of the open balls of given radii r_1 and r_2 about given points $p_1 = (x_1, y_1)$ and $p_2 = (x_2, y_2)$ in $\mathbb{R}^2$. You will need some case distinction. (3 credits)

Exercise 8: Let X be an arbitrary set. Define a distance function $d : X \times X \to \mathbb{R}_0^+$ such that (X, d) is a metric space. (1 credit)

Exercise 9: Define at least 3 different distance functions $d : \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}_0^+$ such that $(\mathbb{R}^n, d)$ is a metric space. (2 credits)

Exercise 10: Let X be a set, and define a distance function $d(x, y)$ on X by setting

$$d(x,y) \ := \begin{cases} 0 & \text{if } x = y, \\ 1 & \text{if } x \neq y. \end{cases}$$

The metric space obtained in this way is called the *discrete metric space*.

1. Which are the open and the closed sets in a discrete metric space?
2. Give a necessary and sufficient criterion for the convergence of a sequence of points in a discrete metric space.

(2 credits)

Exercise 11: Let $X = \{x, y, z\}$ be a set with 3 elements. Find all possible distance functions $d : X \times X \to \{0, 1, 2, 3\}$ endowed with which X becomes a metric space. (2 credits)