Introduction to Topology, Exercise Sheet 4

Stefan Kohl

November 16, 2009

Due: November 30, 2009

Exercise 16: Let $X := \{1, 2, 3\}$. Enumerate all possible topologies on X, and determine equivalence classes with respect to homeomorphism. – How many topologies on X are there up to homeomorphism? (2 credits)

Exercise 17: Find out whether every sequence of points in $\mathrm{SL}(2, \mathbb{R}) \subset \mathbb{R}^4$ endowed with the usual topology has a convergent subsequence. – If yes, give a proof, if not, give a counterexample. (2 credits)

Exercise 18: Let X be an arbitrary set. Define a topology on X such that every sequence of points of X is convergent. (1 credit)

Exercise 19: Define a topology on $\mathbb{Z}$ by letting the open sets be the empty set and the set-theoretic unions of residue classes. Show that $\mathbb{Z}$ with this topology satisfies the axioms for a topological space, and determine whether it is a Hausdorff space or not. (2 credits)

Exercise 20: Use the topology defined in Exercise 19 to show that there are infinitely many prime numbers. – *Hint:* Show that if there would be only finitely many prime numbers, the set $\{-1, 1\}$ would be open, and hence a union of residue classes. (2 credits)

Exercise 21: Give an example of a non-identity homeomorphism of $\mathbb{Z}$ endowed with the topology defined in Exercise 19. (1 credit)