Introduction to Topology, Exercise Sheet 5

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Due: December 14, 2009

Exercise 22: Determine the closure and the interior of $\mathbb{R}$ in $\mathbb{C}$, with respect to the usual topology. (1 credit)

Exercise 23: Let (a_i) be some enumeration of the rationals. Determine the closure and the interior of the set

$$S := \bigcup_{i=1}^{\infty}]a_i - 2^{-i}, a_i + 2^{-i}[\subset \mathbb{R},$$

with respect to

1. the usual topology,
2. the Zariski topology.

(2 credits)

Exercise 24: Let X_1 and X_2 be topological spaces with three points labelled by 1, 2 and 3, where the open sets in X_1 are $\emptyset$, $\{1\}$, $\{1,2\}$ and X_1, and the open sets in X_2 are $\emptyset$, $\{1\}$, $\{2\}$, $\{1,2\}$, $\{1,3\}$ and X_2. Determine the collection of open sets in the product topology on $X_1 \times X_2$. How many open sets are there? (4 credits)

Exercise 25: Let X_1 and X_2 be the topological spaces defined in Exercise 24. Determine all identification maps from X_1 to X_2. (1 credit)

Exercise 26: Let $\mathbb{Z}$ be endowed with the topology defined in Exercise 19, let $r + m\mathbb{Z}$ be a residue class and let $q : \mathbb{Z} \to r + m\mathbb{Z}$, $n \mapsto mn + r$ be an identification map. Determine the collection of open sets in the corresponding quotient topology on $r + m\mathbb{Z}$, and find out whether $\mathbb{Z}$ and $r + m\mathbb{Z}$ with the given topologies are homeomorphic or not. (2 credits)