# Introduction to Topology, Exercise Sheet 6

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Exercise 27: Let $X$ be an arbitrary set. Define a topology on $X$ such that $X$ is a compact topological space. (1 credit)

Exercise 28: In Exercise 16, you have determined all possible topological spaces with 3 points up to homeomorphism. Find out which of them are compact and which are not. (1 credit)

Exercise 29: Determine which of the following topological spaces are compact, and give reasons why they are compact or not:

1. $\mathbb{Z}$ with the discrete topology.
2. $\mathbb{Z}$ with the topology defined in Exercise 19.
3. $\mathbb{R}$ with the Zariski topology.
4. $\mathbb{C}$ with the usual topology.
5. The unit sphere $\{(x, y, z) \in \mathbb{R}^3 \mid x^2 + y^2 + z^2 = 1\}$ with the usual topology.
6. $\mathbb{Q}$ with the topology in which the open sets are $\emptyset$, $\mathbb{Q}$ and all sets $S_n$, $n \in \mathbb{N}$, where $S_n$ is the set of rationals whose denominator is $\leqslant n$.

(6 credits)

Exercise 30: Let $S := \{(x, y, z) \in \mathbb{R}^3 \mid x^2 + y^2 + z^2 = 1\}$ be the unit sphere endowed with the usual topology. Either give an example of a continuous function $f : S \to \mathbb{R}$ which is not bounded, or prove that every continuous function $f : S \to \mathbb{R}$ is bounded. (2 credits)