Introduction to Topology, Exercise Sheet 7

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Due: January 29, 2010

Exercise 31: Is a bounded metric space necessarily also totally bounded? – Either prove or give a counterexample. (2 credits)

Exercise 32: Is a complete metric space necessarily compact? – Either prove or give a counterexample. (1 credit)

Exercise 33: Let $X = \{1, 2, 3, 4, 5, 6\}$ be a metric space, where $d(n, n+1) = n$ for $n < 6$, and $d(1, 6) = 6$. Which are the possible diameters of X? (2 credits)

Exercise 34: Determine the diameter of the following sets (with respect to the usual, i.e. Euclidean, metric):

1. $[0, 1]^4 \subset \mathbb{R}^4$.
2. $\{(x_1, \dots x_n) \in \mathbb{R}^n \mid \sum_{i=1}^n |x_i| \leqslant 1\}$.

(2 credits)

Exercise 35: Let $S := \{(a_1, a_2, a_3, \dots) \in \mathbb{R}^\infty \mid a_1, a_2, a_3, \dots \in [0, 1]\}$.

1. Assume that $\mathbb{R}^\infty$ is endowed with the usual, i.e. the Euclidean, metric. Is the set S bounded? – Either prove or disprove. Do you see any problems? – If so, describe!
2. Assume that S is endowed with the metric coming from the supremum norm, i.e. $d((a_1, a_2, \dots), (b_1, b_2, \dots)) = \sup\{|a_1 - b_1|, |a_2 - b_2|, \dots\}$. Is the set S totally bounded? – Either prove or disprove.

(3 credits – 1 for two correct yes-no answers, the other 2 for your argumentation)