MAT 452: Introduction to Algebra II
Exercise Sheet 2

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Due: Wednesday, May 11, 2011

Exercise 1: Let R be an integral domain, and let $a, b \in R$. Prove that the ideals generated by a and b are the same if and only if $b = ua$ for some unit u of R. (4 credits)

Exercise 2: Prove that the sum of a unit and a nilpotent element of a commutative ring is a unit. (4 credits)

Exercise 3: Prove that the ideals $\langle 2, x \rangle$ of $\mathbb{Z}[x]$ and $\langle x, y \rangle$ of $\mathbb{Q}[x, y]$ are not principal ideals. Also determine the factor rings $\mathbb{Z}[x]/\langle 2, x \rangle$ and $\mathbb{Q}[x, y]/\langle x, y \rangle$. (4 credits)

Exercise 4: Find all ring homomorphisms

1. from $\mathbb{Q}$ to $\mathbb{Z}$ and
2. from $\mathbb{Q}^{3\times 3}$ to $\mathbb{Q}^{2\times 2}$.

Give reasons why there are no ring homomorphisms other than those you have listed. (4 credits)

Exercise 5: Determine the (ring-, respectively field-) automorphism groups of $\mathbb{Z}$ and $\mathbb{Q}$. (4 credits)