

# MAT 452: Introduction to Algebra II

## Exercise Sheet 3

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Due: Monday, June 6, 2011

Exercise 1: Determine and draw the subfield lattice of  $\mathbb{F}_{2^{12}}$ . (4 credits)

Exercise 2: Determine the following:

1. The order and the structure of the group of units of  $\mathbb{F}_{3^4}$ .
2. The order and the structure of the automorphism group of  $\mathbb{F}_{7^4}$ .

(4 credits)

Exercise 3: Find the minimal polynomial of  $\sqrt{2} + \sqrt{3} + \sqrt{5}$  over  $\mathbb{Q}$ . (4 credits)

Exercise 4: Factor the polynomial  $x^{12} - 1 \in \mathbb{Q}[x]$  into irreducible polynomials. (4 credits)

Exercise 5: Can you find an integer  $n \geq 3$  such that  $n^{12} - 1$  has not more prime factors in  $\mathbb{Z}$  than the polynomial  $x^{12} - 1$  has in  $\mathbb{Q}[x]$ ? (4 credits)