MAT 452: Introduction to Algebra II
Spring 2011, Midterm 1, Answers

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Answer to Question 1:

1. Only that it is > 1.
2. That it is 2.
3. That it is $\geqslant 4$, and that if it is finite it is even.
4. It can have at most $46 = 1 + 3 + 2 \cdot 3 + 2^2 \cdot 3 + 2^3 \cdot 3$ such elements.
5. This is possible. For example, it may be the group V_4 of order 4.

Answer to Question 2:

1. $G \cong \mathrm{C}_2 \times \mathrm{C}_2 \times \mathrm{C}_{20} \times \mathrm{C}_{360}$.
2. $G \cong \mathrm{C}_2 \times \mathrm{C}_2 \times \mathrm{C}_4 \times \mathrm{C}_5 \times \mathrm{C}_5 \times \mathrm{C}_8 \times \mathrm{C}_9$.

Answer to Question 3: The orders of subgroups of A_5 are 1,2,3,4,5,6,10,12,60. Examples are

- $\langle () \rangle \cong 1$,
- $\langle (1,2)(3,4) \rangle \cong \mathrm{C}_2$,
- $\langle (1,2,3) \rangle \cong \mathrm{C}_3$,
- $\langle (1,2)(3,4), (1,3)(2,4) \rangle \cong \mathrm{V}_4 \cong \mathrm{C}_2 \times \mathrm{C}_2$,
- $\langle (1,2,3,4,5) \rangle \cong \mathrm{C}_5$,
- $\langle (1,2)(3,4), (3,4,5) \rangle \cong \mathrm{S}_3$,
- $\langle (1,2,3,4,5), (1,2)(3,5) \rangle \cong \mathrm{D}_5$,
- $\langle (1,2,3), (1,2,4) \rangle \cong \mathrm{A}_4$ and
- A_5 itself.

Subgroups of order 15, 20 or 30 would have index 4, 3 or 2. This is not possible, since if there would be such group, A_5 would act transitively on the 4, 3 or 2 cosets of that group, and since $|\,\mathrm{A}_5\,| > 4!$ the kernel of that action would be a nontrivial normal subgroup of A_5.

Answer to Question 4:

1. The center is 1.
2. No, this is not possible, since such subgroup would be normal.
3. Yes, this is possible. Take for example $G = \mathrm{A}_6$ and $H = \mathrm{A}_5$.