MAT 452: Introduction to Algebra II
Spring 2011, Midterm 1

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Date and time: Monday, April 18, 2011, 15:00 - 16:00

Question 1: Let G be a group, and let $a, b, c \in G$ be distinct elements of order 2.

1. What can be said about the order of ab?
2. What can be said about the order of a^b?
3. What can be said about the order of the subgroup $H := \langle a, b, c \rangle < G$?
4. How many elements which can be written as products of 4 or less generators a, b, c can the group H have at most?
5. Is it possible that H has fewer such elements, and why or why not?

(5 credits)

Question 2: Let $G := \mathrm{C}_8 \times \mathrm{C}_{10} \times \mathrm{C}_{18} \times \mathrm{C}_{20}$.

1. Write G in elementary divisors form.
2. Write G as a product of as many as possible direct factors.

(5 credits)

Question 3: Which are the orders of the subgroups of the alternating group A_5? Give an example of a subgroup for each of the possible orders, and give reasons why the other divisors of $|\mathrm{A}_5| = 60$ are not orders of subgroups of A_5.
(5 credits)

Question 4: Let G be a nonabelian finite simple group.

1. Determine the center of G.
2. Is it possible that there is a proper divisor $n > 1$ of the order of G such that G has precisely one subgroup of order n? – Either give an example or give reasons why this cannot be.
3. Is it possible that G has a proper subgroup H which is a nonabelian simple group as well? – Either give an example of such groups G and H or show that this cannot be.

(5 credits)