

MAT 452: Introduction to Algebra II

Spring 2011, Midterm 2, Answers

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Answer to Question 1: Equalities (1.) and (4.) hold for all elements in every ring – the former by commutativity of addition and the latter by left distributivity, while the others do not hold in general.

Answer to Question 2: Subsets (1.), (4.) and (5.) are ideals, while Subsets (2.) and (3.) are not. Subset (4.) is equal to $2\mathbb{Z}$, while Subset (5.) equals \mathbb{Z} .

Answer to Question 3: Examples for (1.) – (5.) are respectively:

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}.$$

Answer to Question 4: The ring \mathbb{Z} of integers may serve as an example for (1.), (3.) and (4.), while $\mathbb{Z}^{2 \times 2}$ and \mathbb{Q} are examples for (2.) and (5.), respectively.