

MAT 452: Introduction to Algebra II

Spring 2011, Midterm 2

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Date and time: Monday, May 16, 2011, 15:00 - 16:00

Question 1: Find out which of the following equalities hold for *all* elements a, b, c, d of *every* ring, and which do not:

1. $a + b = b + a$.
2. $a + b = c + d$.
3. $ab = ba$.
4. $ab(c + d) = abc + abd$.
5. $(a + b)^2 = a^2 + 2ab + b^2$.

(5 credits)

Question 2: Find out which of the following subsets of \mathbb{Z} are ideals of \mathbb{Z} and which are not:

1. $\{0\}$.
2. $\{1\}$.
3. $\{0, 1\}$.
4. $\{2n \mid n \in \mathbb{Z}\}$.
5. $\{3a + 4b \mid a, b \in \mathbb{Z}\}$.

(5 credits)

Question 3: In the ring $\mathbb{Z}^{2 \times 2}$ of 2×2 matrices with integer entries, give an example of

1. a unit,
2. an idempotent,
3. a nilpotent element,
4. an element of order 2, and
5. an element of infinite order.

(5 credits)

Question 4: Give an example of

1. a commutative ring,
2. a noncommutative ring,
3. a principal ideal domain,
4. a unique factorization domain and
5. a field.

(5 credits)