MAT 551: Algebra I
Exercise Sheet 1

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Exercise 1: Let G be a group and let $H < G$ be a subgroup. Further let $a, b \in G$. Prove that the cosets aH and bH are either equal or disjoint.
(2 credits)

Exercise 2: Show that the center of a group is a normal subgroup. (2 credits)

Exercise 3: Show that the set of subgroups of a group is closed under taking intersections, but that it is in general not closed under taking unions. Thus, given a group G and subgroups $H_1, H_2 < G$, show that $H_1 \cap H_2 < G$, but in general not $H_1 \cup H_2 < G$. (2 credits)

Exercise 4: Let G be a group and let $g \in G$ be an element.

1. Assume that g has order $n \in \mathbb{N}$. Given a divisor d of n, show that G has an element of order d.
2. Assume that g has infinite order. What can be said about existence and number of elements of G of given finite order $n \in \mathbb{N}$?

Reminder: the order of a group element $g \in G$ is the least positive integer n such that $g^n = 1$ in case such n exists, and ∞ otherwise. (2 credits)