MAT 551: Algebra I
Exercise Sheet 3

Stefan Kohl

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Due: Wednesday, May 18, 2011

Exercise 11: Prove that the following groups are pairwise nonisomorphic:

1. The additive group $(\mathbb{Z}, +)$ of the integers.
2. The additive group $(\mathbb{Q}, +)$ of the rationals.
3. The multiplicative group $(\mathbb{Q} \setminus \{0\}, \cdot)$ of the rationals.
4. The multiplicative group $(\mathbb{R} \setminus \{0\}, \cdot)$ of the reals.

(4 credits)

Exercise 12: Let G be a group. The *commutator* $[a, b]$ of two elements $a, b \in G$ is defined by $a^{-1}b^{-1}ab$, and the *derived subgroup* G' of G is defined as the subgroup of G which is generated by the set of all commutators of elements of G. Prove that the derived subgroup of G is *characteristic*, i.e. (setwise) invariant under every automorphism of G. (2 credits)

Exercise 13: Let G be a group. Recall that a sequence $G = G_0 \rhd G_1 \rhd G_2 \rhd \cdots \rhd 1$ is called a *subnormal series*. The sequence is called a *normal series* if all subgroups G_i are normal in G. Show by giving a counterexample that a subnormal series is not always a normal series. (2 credits)

Exercise 14: Let G be a group which is generated by 4 distinct elements of order 2 whose product is 1. Prove that G is not simple. (4 credits)

Exercise 15: An automorphism α of a group G is called an *inner* automorphism if there is an element $a \in G$ such that it is given by $\alpha : G \to G,\ g \mapsto g^a$. For $n \neq 6$ the symmetric group S_n has only inner automorphisms, i.e. for $n \notin \{2, 6\}$ the automorphism group $\mathrm{Aut}(\mathrm{S}_n)$ of S_n is isomorphic to S_n itself. Find an automorphism of S_6 which is *not* inner. (4 credits)