MAT 551: Algebra I
Spring 2011, Final Exam

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Date and time: Monday, June 27, 2011, 14:00 - 16:00

Question 1: Compute the following:

1. The product $(7,8) \cdot (6,7) \cdot (5,6) \cdot (4,5) \cdot (3,4) \cdot (2,3) \cdot (1,2)$.
2. The sign of the permutation $(1,2)(3,4,5)(6,7,8,9)$.
3. The order of the group $G := \langle (1,2)(3,4)(5,6), (1,3,5)(2,4,6) \rangle < \mathrm{S}_6$.
4. The order of the group $\mathrm{GL}(2,4)$.
5. The order of the group $\mathrm{PSL}(2,7)$.
6. The center of the group $\mathrm{SL}(3,\mathbb{Z})$.

(10 credits – 1 each for (1.)-(2.), and 2 each for (3.)-(6.))

Question 2: Compute the following:

1. The order of the direct product $\mathrm{A}_4 \times \mathrm{D}_4$.
2. The smallest n such that $\mathrm{A}_4 \times \mathrm{D}_4$ embeds into S_n.
3. The index of $\mathrm{A}_4 \times \mathrm{D}_4$ in S_n for the n found in (2.).
4. A Sylow 2-subgroup of $\mathrm{A}_4 \times \mathrm{D}_4$.
5. All possible orders of semidirect products $\mathrm{A}_4 \rtimes \mathrm{D}_4$.

(10 credits – 2 each for (1.)-(5.))

Question 3: Determine a subgroup $G < \mathrm{S}_7$ of order 21. (6 credits)

Question 4: Let

$$G \ := \ \langle (1,2,3,4,5,6,7,8,9,10,11,12), (2,4)(6,8)(10,12), (1,3,5,7) \rangle \ < \ \mathrm{S}_{12}.$$

Find out whether the action of G on the set $S := \{1,2,3,4,5,6,7,8,9,10,11,12\}$ is primitive. If it is primitive, find out whether it is 2-transitive. Otherwise find a block system, i.e. an equivalence relation on S which is invariant under the action of G. (6 credits – proof required!)

Question 5: Let G be a group which is generated by four pairwise distinct elements a, b, c and d of order 2. Further let $H < G$ be the subgroup which is generated by all products of 2 generators, i.e. by ab, ac ad, bc, bd and cd. Determine all possible values of the index $[G : H]$ of H in G, and give representatives for the cosets of H in G in all cases. (8 credits – proof required!)