MAT 551: Algebra I
Spring 2011, Midterm 1

Stefan Kohl

Date and time: Monday, May 2, 2011, 16:30 - 17:45

Question 1:

1. In $(\mathbb{Z}, +)$ compute $1 + 1$ and 3^2. For the last expression give both possible interpretations. (Be careful – the group $(\mathbb{Z}, +)$ is not the ring $\mathbb{Z}$, so there is no '$\cdot$'.)
2. In S_6 compute $(1, 2) \cdot (1, 3)$, $(1, 2, 3, 4, 5)^{(2,3)(4,5)}$ and $((1, 2, 3, 4)(5, 6))^{12}$.

(6 credits)

Question 2: Find out which of the following abelian groups are isomorphic to $\mathrm{C}_4 \times \mathrm{C}_6 \times \mathrm{C}_8$ and which are not:

1. $\mathrm{C}_2 \times \mathrm{C}_4 \times \mathrm{C}_{24}$.
2. $\mathrm{C}_2 \times \mathrm{C}_6 \times \mathrm{C}_{16}$.
3. $\mathrm{C}_2 \times \mathrm{C}_3 \times \mathrm{C}_4 \times \mathrm{C}_8$.
4. $\mathrm{C}_2 \times \mathrm{C}_8 \times \mathrm{C}_{12}$.

(4 credits)

Question 3: Let G be a group which is generated by four pairwise distinct elements a, b, c and d of order 2.

1. Either compute the order of G or explain why the given information is not enough for this.
2. Can you give a reason why the group G is not simple?
3. How many elements which can be written as products of 3 or less of the generators a, b, c, d can the group G have at most?

(6 credits)

Question 4: Let $G < \mathrm{S}_{11}$ be a group which acts 4-transitively on the set $\{1, 2, \ldots, 11\}$ and in which no element except for the identity moves less than 8 points. Compute the order of G. (4 credits, +2 extra credits if you can tell the name of the group G – it's famous)