MAT 551: Algebra I
Spring 2011, Midterm 2

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Date and time: Wednesday, May 25, 2011, 16:30 - 17:45

Question 1: Compute the following:

1. The product $(1,2,3)(4,5) \cdot (1,2,3,4,5)$.
2. The power $((1,2,3,4)(5,6,7))^5$.
3. The conjugate $(1,2,3,4,5)^{(3,4,5)}$.
4. The order of the permutation $(1,2,3)(4,5)(6,7,8,9,10)$.
5. The sign of the permutation $(1,2,3)(4,5,6)$.
6. The sign of the permutation $(1,2,3,4)(5,6)(7,8)$.

(6 credits)

Question 2: Compute the following:

1. The order of the symmetric group S_4 of degree 4.
2. The order of the alternating group A_5 of degree 5.
3. The index of the Klein 4-group V_4 in S_4.
4. The number of conjugacy classes of S_5.
5. The number of conjugacy classes of elements of order 2 in S_6.
6. The number of Sylow 2-subgroups of S_4.

(6 credits)

Question 3: Let $n \in \mathbb{N}$, and let $G \leqslant \mathrm{S}_n$ be a group which acts 2-transitively on the set $\{1, \ldots, n\}$ and which contains the transposition $(1,2)$. Determine the group G. (4 credits – 1 for the result, 3 for the proof)

Hint: look at the conjugates of $(1,2)$ in G.

Question 4: Prove that there is no simple group of order 120. (4 credits)

Hint: assume that there would be a simple group G of order 120. For a suitable prime divisor p of $|G|$, consider the action of the group G on the set of its Sylow p-subgroups via conjugation.