Midterm Test in MAT 641: Computational Algebra I

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Date and time: Tuesday, November 24, 2009, 14:00 - 15:00.
Room: C 411

NAME:

You have **60 minutes** of time to answer the 12 questions below. Proofs are only required when questions explicitly ask for them. You are not allowed to use anything else than a pen and blank sheets of paper. Keep your mobile phones switched off during the examination. You may write on extra blank sheets of paper, if the space provided after a question is not sufficient for your solution. If you do so, be sure to label your solutions with the numbers of the respective questions. Also, write in the space after the question "see extra sheet of paper" etc. Do not forget to write your **name** on **every** sheet of paper you return to me! If you attempt to cheat, you will fail this test – further consequences are possible.

Question 1: Give the definition of a *field*. (3 credits)

Question 2: Give the definition of the *characteristic* of a field. (2 credits)

Question 3: Precisely for which positive integers n is there a finite field with n elements? (1 credit)

Question 4: Why is it not necessary to distinguish between left and right ideals in polynomial rings? (2 credits)

Question 5: When exactly is the polynomial ring $K[x_1, \ldots, x_n]$ in n variables over a field K a principal ideal domain? (2 credits)

Question 6: Give an example of a polynomial $f \in \mathbb{F}_3[x] \setminus \{0\}$ such that $f : \mathbb{F}_3 \to \mathbb{F}_3$ is the zero function. (2 credits)

Question 7: Give the definition of an *affine variety*. (2 credits)

Question 8: Give the definition of a *rational parametric representation* of an affine variety. (4 credits)

Question 9: When is an affine variety called *unirational*? (2 credits)

Question 10: Describe the location of a *Bezier cubic* relative to its control polygon. (2 credits)

Question 11: Is the union of infinitely many affine varieties necessarily also an affine variety? – Either prove or disprove. (5 credits)

Question 12: Let V be the affine variety $\langle x^4 + y^4 - 1 \rangle \subset \mathbb{Q}^2$. Determine $|V|$. (No proof required!) (3 credits)

– Good luck!

Maximum possible number of credits: 30.
Grade = (number of credits)/3, rounded to the nearest integer.