Computational Algebra II, Exercise Sheet 2

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Due: April 29, 2010

Exercise 1: Determine all points with integer coordinates on the following affine varieties:

1. $V(x^2 + y, x + y^2)$. (2 credits)
2. $V(4x^2 - 45y^2 + 4x - 30y - 5)$. (4 credits)

Exercise 2: Let $I := \langle xy^3, x^2y^2, x^3y \rangle \subset \mathbb{R}[x, y]$ be an ideal. Either prove or disprove that $\{xy^3, x^2y^2, x^3y\}$ is a Groebner basis for I. (2 credits)

Exercise 3: Let $I \subset \mathbb{R}[x_1, \dots, x_n]$ be an ideal, and let $B = \{f_1, \dots, f_m\} \subset I$ be a Groebner basis for I. Either prove or disprove the following assertions:

1. The set $\tilde{B} := \{f_m, f_{m-1}, \dots, f_1\} \subset I$ is a Groebner basis for I as well. (2 credits)
2. For every $f_{m+1} \in I$, the set $\tilde{B} := \{f_1, \dots, f_m, f_{m+1}\}$ is a Groebner basis for I as well. (2 credits)

Exercise 4: Let $I := \langle x^2 + y, x + y^2 \rangle \subset \mathbb{R}[x, y]$ be an ideal. Assume lex order with $x > y$.

1. Find a Groebner basis for I. (4 credits)
2. How many elements has a Groebner basis for I at least? (2 credits)
3. Find a Groebner basis for I with the smallest possible number of elements. (2 credits)