Computational Algebra II, Exercise Sheet 3

Stefan Kohl

May 1, 2010

Due: May 18, 2010

Exercise 1: Is it true that no basis of a polynomial ideal can be shorter (i.e. have fewer elements) than its reduced Groebner basis? – Either prove or give a counterexample. (4 credits)

Exercise 2: Determine all points (i.e. not only those with integer coordinates, etc.) on the affine variety $V(x^2 + y, y^2 + z, z^2 + x) \subset \mathbb{C}^3$. (6 credits)

Exercise 3: Find out which of the following ideals of $\mathbb{R}[x, y, z]$ are equal (if any):

$$\begin{aligned}
I_1 &= \langle x + y + z - 1\rangle, \\
I_2 &= \langle x + y + z - 1, x^2 + y^2 + z^2 - 1\rangle, \\
I_3 &= \langle x + y + z - 1, x^2 + y^2 + z^2 - 1, x^3 + y^3 + z^3 - 1\rangle, \\
I_4 &= \langle x + y + z - 1, x^2 + y^2 + z^2 - 1, x^3 + y^3 + z^3 - 1, x^4 + y^4 + z^4 - 1\rangle, \\
I_5 &= \langle x + y + z - 1, x^2 + y^2 + z^2 - 1, x^3 + y^3 + z^3 - 1, x^4 + y^4 + z^4 - 1, x^5 + y^5 + z^5 - 1\rangle.
\end{aligned}$$

(4 credits)

Exercise 4: Let a, b and c be real numbers which satisfy the equations

$$\begin{aligned}
a + b + c &= 3, \\
a^2 + b^2 + c^2 &= 5, \\
a^3 + b^3 + c^3 &= 7.
\end{aligned}$$

Verify that $a^4 + b^4 + c^4 = 9$, and compute $a^5 + b^5 + c^5$, $a^6 + b^6 + c^6$ and $a^7 + b^7 + c^7$.
(6 credits)