Final Test in MAT 642: Computational Algebra II

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Date and time: Tuesday, June 22, 2010, 14:00 - 16:00.
Name:

You have **120 minutes** of time to answer the 8 questions below. Write your answers to the blank sheets of paper supplied to you. You are not allowed to use anything else than a pen.

Question 1: Give the definitions of a

- *basis*, of a
- *Groebner basis*, of a
- *minimal Groebner basis* and of a
- *reduced Groebner basis*

of an ideal of a polynomial ring, and explain the importance of these four terms. (8 parts, worth 2 credits each – 16 credits in total)

Question 2: State the *Buchberger algorithm*. (6 credits)

Question 3: Explain how to solve a system of polynomial equations by computer. Is it always a good idea to proceed in the same way in hand computations? – Explain. (6 credits)

Question 4: Find all solutions $(x, y, z) \in \mathbb{C}^3$ of the following system of polynomial equations:

$$\begin{aligned} x^2y + 2yz - z^3 &= 1 \\ x^3 - 2x^2y + 2z^3 &= 2 \\ \tfrac{1}{4}x^3 + yz &= 3 \end{aligned}$$

(6 credits)

Question 5: Find all solutions $(x, y, z) \in \mathbb{C}^3$ of the following system of polynomial equations:

$$\begin{aligned} x + y + z &= 0 \\ x^2y + 2xy^2 + xyz + y^3 + y^2z &= 1 \end{aligned}$$

(6 credits)

Question 6: Try to determine all solutions $(x, y) \in \mathbb{C}^2$ of the following system of polynomial equations:

$$\begin{aligned} x^4 - y &= 1 \\ x - y^4 &= 1 \end{aligned}$$

Which problem do you encounter when trying to write down the solutions? (6 credits)

Question 7: Compute the reduced Groebner bases for the ideal $\langle x^2 + y - 1, xy + y - 1\rangle \subset \mathbb{C}[x, y]$ for

1. lex order and
2. grlex order,

where $x > y$. (6 credits)

Question 8: Is there a constant c such that all ideals of $\mathbb{C}[x, y]$ can be generated by c or fewer polynomials? – Either prove or disprove. (8 credits)

– Good luck!

Maximum possible number of credits: 60.