Midterm 2 in MAT 642: Computational Algebra II

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Date and time: Tuesday, May 25, 2010, 14:00 - 15:00.
Room: C 411

NAME:

You have **60 minutes** of time to answer the 7 questions below. You are not allowed to use anything else than a pen and blank sheets of paper.

Question 1: When is a Groebner basis said to be *minimal*, and when is it said to be *reduced*? (4 credits)

Question 2: Explain the importance of the concept of a *reduced* Groebner basis. (3 credits)

Question 3: Let $n \in \mathbb{N}$, let $r \in \{0, \ldots, n\}$ and let K be some field. Give the definition of the r-th *elimination ideal* I_r of an ideal I of $K[x_1, \ldots, x_n]$. (3 credits)

Question 4: State the *Elimination Theorem*. (4 credits)

Question 5: Briefly explain why the *Buchberger Algorithm* terminates. (4 credits)

Question 6: Compute reduced Groebner bases for the following ideals for lexicographic order:

1. $\langle a, b, c, d, e, f\rangle \subset \mathbb{C}[a, b, c, d, e, f]$.
2. $\langle x^2y^3z^2 + \sqrt{2}xy^2 - 5x^2y + 7, 8 + \sqrt{5}\pi^2, x^2y^2z - 3x^3y + 2xyz^2\rangle \subset \mathbb{C}[x, y, z]$.

(4 credits)

Question 7: Solve the system of equations

$$\begin{aligned} x^2 + y^2 &= 1, \\ xy + x + y &= 1 \end{aligned}$$

for $x, y \in \mathbb{C}$. – All solutions need to be found. (8 credits – 2 for the trivial and 6 for the nontrivial solutions)

– Good luck!

Maximum possible number of credits: 30.
Grade = (number of credits)/3, rounded to the nearest integer.